COMPLETE AND STABLE PROJECTIVE HARMONIC INVARIANTS FOR PLANAR CONTOURS RECOGNITION

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Keywords: Projective invariance, descriptors, completeness, stability, planar contours, shape recognition.

Abstract: Planar shapes recognition is an important problem in computer vision and pattern recognition. We deal with planar shape contour views that differ by a general projective transformation. One method for solving such problem is to use projective invariants. In this work, we propose a projective and parameterization invariant generation framework based on the harmonic analysis theory. In fact, invariance to reparameterization is obtained by a projective arc length curve reparameterization process. Then, a complete and stable set of projective harmonic invariants is constructed from the Fourier coefficients computed on the reparameterized contours. We experiment this set of descriptors on analytic contours in order to recognize projectively similar ones.

1 INTRODUCTION

The recognition of planar shapes that are subjected to certain viewing transformations has increasing interest in many computer vision applications such as robotic vision, data-base retrieval, registration and three-dimensional (3D) reconstruction. Three dimensional objects could be also considered as planar when the camera is far away from the object and distances within the object are negligible. Planar shapes are generally assumed to have a piecewise smooth boundary that is represented by a bidimensional (2D) continuous contour. When a contour undergoes rigid motion and is then projected onto an image plane using a pinhole camera, the perspectively projected contour image can be represented by a planar projective transformation.

The use of projective-invariant approach to deal with planar shape recognition problem in different views seems to be the most efficient method mainly when camera parameters or point-to-point correspondences are unknown. In fact, a projective invariant is a property of geometric configurations in one view which remain unchanged under the projective transformations (Mundy and Zisserman, 1992). In the planar case, projective transformations also called plane-to-plane homographies have the structure of a group. This group includes the well known Euclidean and Affine groups.

Two main classes of planar projective invariants have been studied : algebraic and differential invariants. The algebraic invariants were applied to algebraic objects such as points, lines and conics. The well known algebraic invariant is cross-ratio (Mundy and Zisserman, 1992). Algebraic invariants are often global and deal with the whole shape. However, it's generally hard to fit polynomials to complex shapes. Differential invariants are applied to smooth curves. They are based on local properties of shapes such as derivatives however they require generally high order derivatives (Weiss, 1992; Van Gool et al., 1992). Furthermore, as the invariants are local, the local correspondence between points of the images obtained from different viewpoints should be known. Thus, differential invariants cannot be applied directly and needs other methods in order to solve their problems. The semi-differential invariants has been introduced to reduce the order of derivatives by adding reference points (Brill et al., 1992; Van Gool et al., 1992). In addition, integral features approach integrates the local invariants over the original arbitrary curve parameter and provides global or integral invariants such as moments. Recently, a class of integral invariants with respect to the Euclidean group are proposed in (Manay et al., 2006). This set of invariants allows the analysis of shapes at multiple scales.

Furthermore, Fourier analysis theory has provided curves global invariants in the Euclidean case (Fourier Descriptors) (Crimmins, 1982; Kunttu et al., 2004) and affine case (Arbter et al., 1990). In (Kuthirummal et al., 2004), the authors have proposed an algebraic affine recognition constraint.

Although, differential invariants remain constant in the case of projectivities, they still generally depend on the curve parameterization. The parameterization is chosen arbitrary and would not be necessary the same for different views. Thus, we need to deal with both parameterization and geometric transformation invariance. Some works have consider such problem and have proposed projective invariant descriptors which are independent with respect parameterization (Weiss, 1992; Van Gool et al., 1992).

In this paper, we propose a projective and parameterization invariant generation framework based on the harmonic analysis theory and differential geometry. In fact, we perform a projective curve reparameterization with a projective arc length. Thus two equiprojective reparameterized contours from two different views are equivalent up to a starting point. Then, a complete and stable set of projective harmonic invariants is introduced by computing the $\mathbb{C}^3$-Fourier coefficients on projective arc length reparameterized contours.

The next section characterises the transformation in the case of a projection by a pinhole camera. Then, the equiprojective reparameterization process is introduced. In section 4, we construct the complete and stable set of projective invariants. Next, the NURBS curve fitting is introduced. Section 6 presents some experimental results.

2 GEOMETRIC TRANSFORMATION AND PERSPECTIVE PROJECTION

To characterize the geometric transformation between two corresponding shape contours, we review the concept of planar projective homography. Planar projective homography (also called projectivity) is a linear mapping in the planar projective space $\mathbb{P}^2$, $\mathcal{H} : \mathbb{P}^2 \to \mathbb{P}^2$ defined up to an arbitrary factor λ by a 3×3 matrix $\mathbf{H}$.

The relation between corresponding views of points on a world plane Π in a 3D space, can be modeled by a planar homography induced by the plane. Consider two views p and p' of a 3D space point $P \in \Pi$, in two camera frames f and f' respectively. We will denote their corresponding homogeneous coordinates by $\widetilde{p} = (x, y, 1)$, $\widetilde{p'} = (x', y', 1)$ and $\widetilde{P} = (X, Y, 1)$. Let $M = K[I|0]$ and $M' = K'[R|t]$ be the first and the second camera projection matrices (respectively), where R and t are the relative rotation and translation between the cameras and K and K' are the respective internal calibration matrices. Thus, $\widetilde{p} = K[I|0]\widetilde{P}$ and $\widetilde{p'} = K'[R|t]\widetilde{P}$.

Let n be the unit normal vector to the plane Π and let $d > 0$ denote the distance of Π from the optical center of the first camera. The linear transformation from $\widetilde{p}$ to $\widetilde{p'}$ can be expressed as:

$$\widetilde{p'} = K'(R + \frac{1}{d} t n^T) K^{-1} \widetilde{p} = \mathbf{H}\widetilde{p}$$

3 G-INVARIANT REPARAMETERIZATION

It was proven in differential geometry that a simple curve is homeomorphic to the unit circle S^1 or the real line $\mathbb{R}$. Here, we consider only the first case which corresponds to closed contours. Thus, planar shapes are represented by their smooth boundaries as a closed 2D continuous parametric curve. In homogenous coordinates, a parameterization $\gamma(t)$ of a planar curve γ is an 1-periodic function of a continuous parameter t defined by:

$$\begin{array}{rccl} \gamma: & [0,1] & \longrightarrow & \mathbb{R}^3 \\ & t & \longmapsto & \gamma(t) = [x(t), y(t), 1]^t. \end{array} \quad (1)$$

and noted by $\gamma(t)$.

Throughout this section, we indicate with $\gamma : S^1 \to \mathbb{R}^2$ a closed planar contour and G a group acting on $\mathbb{R}^2$.

It's well known that a same parametric curve may have different parameterizations. The invariants computed from two different parameterizations of the same geometric curve are generally different. This is due to parameterization dependence on transformations. One solution to this problem consists in performing a G-invariant reparameterization of the curve where G is the geometric transformations group.

Definition 3.1. *A reparameterization of a curve* γ*, noted* $(\gamma(\widehat{t}))$*, is defined as follows:*

$$\gamma(\tilde{t}) = \gamma(\tau(t)) = [x(\tau(t)), y(\tau(t))]^t, \; t \in [0,1]. \quad (2)$$

where τ *is an increasing function defined on* $[0,1]$.

Definition 3.2. *A $\mathbb{G}$-invariant reparameterization is the process of reparameterizing the curve by a G-invariant arc length.*

Let $\gamma_1(t_1)$ and $\gamma_2(t_2)$ two parameterizations of a geometric curve and its image by a geometric transformation $\mathbf{g}$. After $\mathbb{G}$-invariant reparameterization, both curve parameterizations verify the following equation :

$$\gamma_2(\tilde{t}) = \mathbf{g}(\gamma_1(\tilde{t}+t_0)),\ t_0 \in \mathbb{Z} \text{ et } \mathbf{g} \in \mathbb{G}, \quad (3)$$

where t_0 is departure points difference between the contours.

Here we study the case of planar projective transformations. Many projective arc lengths have been introduced in literature. The classical projective arc length is defined by the following equation (Cartan, 1937).

$$\sigma(t) = \frac{1}{L}\int_0^t \sqrt[3]{|H(u)|}du,\ t \in [0,1] \quad (4)$$

where

$$H(t) = -\frac{1}{3}pq + \frac{2}{27}p^3 - \frac{1}{2}q' + \frac{1}{3}pp' + \frac{1}{6}p'',$$

$$p = -\frac{\det(\gamma'''(t)\ \gamma'(t))}{\det(\gamma''(t)\ \gamma'(t))},\ q = \frac{\det(\gamma'''(t)\ \gamma''(t))}{\det(\gamma''(t)\ \gamma'(t))}$$

and L is the curve projective length given by:

$$L = \int_0^1 \sqrt[3]{|H(u)|}du.$$

4 PROJECTIVE HARMONIC INVARIANTS

Let two planar curves γ_1 and γ_2 projectively similar. After $PGL(2)$-invariant reparameterization, these curves given by their homogenous coordinates verify the following equation:

$$\gamma_2(\tilde{t}) = \mathbf{H}\gamma_1(\tilde{t}+t_0),\ t_0 \in \mathbb{Z}, \quad (5)$$

where $\mathbf{H}$ is the planar projective transformation matrix and t_0 is the departure point difference between the two curves. We recall that a parametric representation of a planar curve is an 1-periodic function of a continuous parameter t. Thus, Fourier coefficients of the two curve reparameterizations exist and they are related by:

$$c_n[\gamma_2] = e^{2i\pi n t_0}\mathbf{H}c_n[\gamma_1],\ \forall n \in \mathbb{N}, \quad (6)$$

where $c_n[\gamma_i]$ are Fourier coefficients of γ_i.

Thus, $PGL(2)$-invariant descriptors of a curve γ could be constructed as follows :

$$I^1_{k_0} = |\Delta^1_{k_0,k_2,k_3}|,\ I^1_{k_1} = |\Delta^1_{k_1,k_2,k_3}|,$$
$$I^1_{k_2} = |\Delta^1_{k_2,k_0,k_1}|,\ I^1_{k_3} = |\Delta^1_{k_3,k_0,k_1}|,$$

for all $k \in \mathbb{N}\backslash\{k_0,k_1,k_2,k_3\}$,

$$I^1_k = \Delta^{k_2-k_3}_{k,k_0,k_1}\Delta^{k_3-k}_{k_0,k_1,k_2}\Delta^{k-k_2}_{k_0,k_1,k_3}, \quad (7)$$

where $\Delta^p_{k,l,m} = \det(c_k[\gamma],c_l[\gamma],c_m[\gamma])^p$ and $\det(x_1,x_2,x_3)$ denotes the determinant of a matrix which consists of three column vectors x_1, x_2 and $x_3 \in \mathbb{R}^3$.

4.1 Invariance

In this section we demonstrate the homography invariance of the proposed set of invariants. We consider a parametric curve γ and its image γ_t by an homography transformation $\mathbf{H}$. Let $\mathbf{M}(\gamma) = [c_k[\mathbf{C}],c_l[\gamma],c_m[\gamma]]$ and $\mathbf{M}(\gamma_t) = [c_k[\mathbf{C}_t],c_l[\gamma_t],c_m[\gamma_t]]$ respectively the matrices composed by the k^{th}, l^{th} and m^{th} fourier coefficient rows of γ and γ_t.

Let $\Delta^p_{k,l,m}(\gamma) = \det(\mathbf{M}(\gamma))^p$ where det(.) is the determinant operator. Thus,

$$\Delta^p_{k,l,m}(\gamma_t) = e^{2inp(k+l+m)}|\det(\mathbf{H})|^p\Delta^p_{k,l,m}(\gamma) \quad (8)$$

The descriptor set of the transformed curve γ_t is then given by :

$$\begin{aligned} I^1_k(\gamma_t) = &\ e^{2in[(k_2-k_3)(k+k_0+k_1)+(k_3-k)(k_0+k_1+k_2)+\cdots} \\ &(k-k_2)(k_0+k_1+k_3)] \\ = &\ |\det(\mathbf{H})|^{(k_2-k_3+k_3-k+k-k_2)}I^1_k(\gamma) \end{aligned}$$

and

$$\begin{aligned} &(k_2-k_3)(k+k_0+k_1)+(k_3-k)(k_0+k_1+k_2)+ && \cdots \\ &(k-k_2)(k_0+k_1+k_3) && = 0 \\ &k_2-k_3+k_3-k+k-k_2 && = 0 \end{aligned}$$

so $I^1_k(\gamma_t) = I^1_k(\gamma)$.

4.2 Completeness

This set of invariants is not complete. In order to ensure the completeness property, we propose to complete it with the two following sets constructed relatively to two other fixed indices of k_0, denoted by k_4 and k_5 :

$$I^2_{k_4} = |\Delta^1_{k_4,k_2,k_3}|,$$
$$I^2_{k_2} = |\Delta^1_{k_2,k_4,k_1}|,\ I^2_{k_3} = |\Delta^1_{k_3,k_4,k_1}|,$$

for all $k \in \mathbb{N}\backslash\{k_4,k_1,k_2,k_3\}$,

$$I^2_k = \Delta^{k_2-k_3}_{k,k_4,k_1}\frac{\Delta^{k_3-k}_{k_4,k_1,k_2}}{|\Delta^{k_3-k}_{k_4,k_1,k_2}|}\frac{\Delta^{k-k_2}_{k_4,k_1,k_3}}{|\Delta^{k-k_2}_{k_4,k_1,k_3}|}, \quad (9)$$

$$I^3_{k_5} = |\Delta^1_{k_5,k_2,k_3}|,$$

$$I^3_{k_2} = |\Delta^1_{k_2,k_5,k_1}|, \; I^2_{k_3} = |\Delta^1_{k_3,k_5,k_1}|,$$

for all $k \in \mathbb{N} \backslash \{k_5, k_1, k_2, k_3\}$,

$$I^3_k = \Delta^{k_2-k_3}_{k,k_5,k_1} \frac{\Delta^{k_3-k}_{k_5,k_1,k_2}}{|\Delta^{k_3-k}_{k_5,k_1,k_2}|} \frac{\Delta^{k-k_2}_{k_5,k_1,k_3}}{|\Delta^{k-k_2}_{k_5,k_1,k_3}|}, \tag{10}$$

where $\Delta^p_{k,l,m} = det(c_k[\gamma], c_l[\gamma], c_m[\gamma])^p$ and $det(x_1, x_2, x_3)$ denotes the determinant of a matrix which consists of three column vectors x_1, x_2 and $x_3 \in \mathbb{R}^3$.

The proof of the completeness property is as follows: We denote by:

$$\begin{array}{ll} \theta^1_2 = Arg(\Delta_{k_2,k_0,k_1}) & \theta^1_3 = Arg(\Delta_{k_3,k_0,k_1}) \\ \theta^2_2 = Arg(\Delta_{k_2,k_4,k_1}) & \theta^2_3 = Arg(\Delta_{k_3,k_4,k_1}) \\ \theta^3_2 = Arg(\Delta_{k_2,k_5,k_1}) & \theta^3_3 = Arg(\Delta_{k_3,k_4,k_1}) \end{array} \tag{11}$$

We obtain the following system of determinants :

$$\begin{cases} \Delta_{k,k_0,k_1} = I^1_k [I^1_{k_2}]^{\frac{k-k_3}{k_3-k_2}} [I^1_{k_3}]^{\frac{k_2-k}{k_3-k_2}} e^{\frac{(k-k_2)\theta^1_2 + (k_3-k)\theta^1_2}{k_3-k_2}} \\ \Delta_{k,k_4,k_1} = I^2_k [I^2_{k_2}]^{\frac{k-k_3}{k_3-k_2}} [I^2_{k_3}]^{\frac{k_2-k}{k_3-k_2}} e^{\frac{(k-k_2)\theta^2_2 + (k_3-k)\theta^2_2}{k_3-k_2}} \\ \Delta_{k,k_5,k_1} = I^3_k [I^3_{k_2}]^{\frac{k-k_3}{k_3-k_2}} [I^3_{k_3}]^{\frac{k_2-k}{k_3-k_2}} e^{\frac{(k-k_2)\theta^3_2 + (k_3-k)\theta^3_2}{k_3-k_2}} \end{cases} \tag{12}$$

Thus, we can reconstruct the Fourier coefficients $c_k[\mathbf{C}]$ once the value of the three determinants are known. So, to determine $c_k[\mathbf{C}]$, we have the equations system :

The unique solution is given by :

$$c_k[\mathbf{C}] = \frac{e^{-i\theta^4_{k_0}}}{I^4_{k_0}} \begin{pmatrix} det(E, c_{k_0}[\mathbf{C}], c_{k_1}[\mathbf{C}]) \\ det(E, c_{k_4}[\mathbf{C}], c_{k_1}[\mathbf{C}]) \\ det(E, c_{k_5}[\mathbf{C}], c_{k_1}[\mathbf{C}]) \end{pmatrix} \tag{13}$$

where

$$\mathbf{E} = \begin{pmatrix} \Delta_{k,k_0,k_1} \\ \Delta_{k,k_4,k_1} \\ \Delta_{k,k_5,k_1} \end{pmatrix} \tag{14}$$

and $\theta^4_{k_0} = Arg(\Delta_{k_0,k_5,k_1})$.

4.3 Stability

The power values $(k - k_2)$ or $(k_3 - k)$ could be negative so the invariant function becomes an hyperbolic function which is not continuous. In order to solve this problem, we propose to divide by the corresponding complex modules. So, we obtain the following stable invariant set :

$$I^1_{k_0} = |\Delta^1_{k_0,k_2,k_3}|, \; I^1_{k_1} = |\Delta^1_{k_1,k_2,k_3}|,$$

$$I^1_{k_2} = |\Delta^1_{k_2,k_0,k_1}|, \; I^1_{k_3} = |\Delta^1_{k_3,k_0,k_1}|,$$

for all $k \in \mathbb{N} \backslash \{k_0, k_1, k_2, k_3\}$,

$$I^1_k = \Delta^{k_2-k_3}_{k,k_0,k_1} \frac{\Delta^{k_3-k}_{k_0,k_1,k_2}}{|\Delta^{k_3-k}_{k_0,k_1,k_2}|} \frac{\Delta^{k-k_2}_{k_0,k_1,k_3}}{|\Delta^{k-k_2}_{k_0,k_1,k_3}|}, \tag{15}$$

where $\Delta^p_{k,l,m} = det(c_k[\mathbf{C}], c_l[\mathbf{C}], c_m[\mathbf{C}])^p$ and $det(x_1, x_2, x_3)$ denotes the determinant of a matrix which consists of three column vectors x_1, x_2 and $x_3 \in \mathbb{R}^3$.

Such development is not a rigorous proof of stability criterion. In future work, we will give a way to establish the stability property.

5 CONTOUR FITTING WITH NURBS

In image analysis, data is always discrete. So approximation or interpolation methods are needed to get a continuous representation of the studied object. In the case of objects described by their external contours, these methods are called curve algorithms. A curve algorithm is invariant to a transformations group G if and only if :

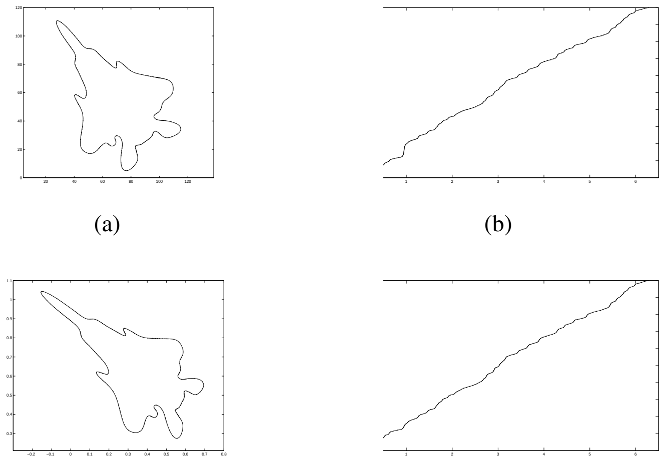
$$g.F(D) = F(g.D) \quad \forall g \in G, \, \forall D \in D^n. \tag{16}$$

This means that applying a curve algorithm to the image by a transformation g of a data set is equivalent to the image of the curve algorithm applied to the discrete data by the same transformation g. The NURBS (*Non-Uniform Rational BSplines*) are curve algorithms invariant to projective transformations.

In this work, we have used the optimal interpolation scheme proposed by (Gaffney and Powell, 1976) since it provides the center function in the band formed by all interpolants to the given data that, in addition, have their k^{th} derivative between $-M$ and M (for large M).

6 EXPERIMENTAL RESULTS

In this section, we present some experiments that illustrate the different steps needed to compute the proposed set of projective invariants. First, we consider two planar contours obtained up on a projective transformation. Figures 1(a) and 1(c) show the projective arc length parameterization of both contours. It's important to notice that the obtained parameters of both contours are upon a translation. The projective arc lengths computed in the reparameterization step are shown in figures 1(b) and 1(d). The performance of the proposed descriptors set is evaluated using a set of 204 analytic contours created by performing planar



(a) (b) (c) (d)

Figure 1: Planar contours reparameterization results.

homographies on a set of six parametric contours 2(a). The planar homographies are obtained by varying intrinsic and extrinsic camera parameters.

Figures 2(b) and 2(c) show shape matching results. The distance used to compute similarity between two contours is the Euclidean one. We notice that the contour $cc-1$ is matched to 21 correct contours. For the case of the contour $cc-4$, 11 contours are correctly matched. The matching errors are due to approximations and the required high order derivatives.

7 CONCLUSION

In this paper we proposed complete and stable projective invariant descriptors. This set of descriptors is based on a projective arc length parameterization. The invariance, completeness and stability of these descriptors are theorically proved. Experimental results of contour reparameterization process are presented. Promising shape matching results are obtained on a set of analytic planar closed contours. In our future work, we intend to test these descriptors on a set of planar contours extracted from grey-level images. Furthermore, the robustness of the proposed descriptors set to noise will be explored.

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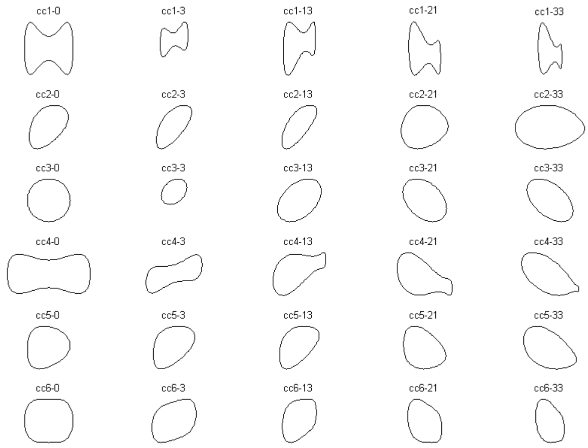
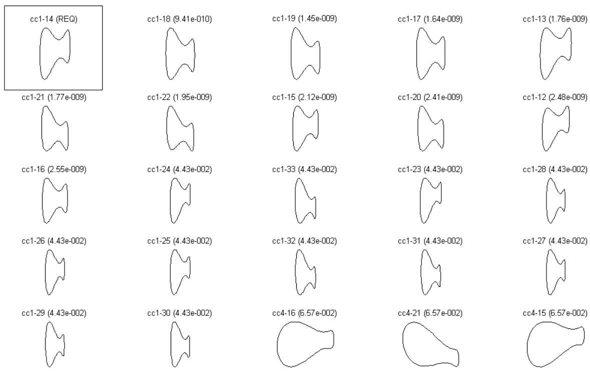
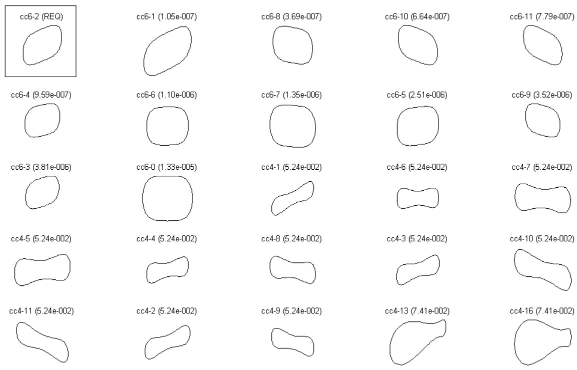


(a)



(b)



(c)

Figure 2: (a) set of planar contours; (b) and (c) shape matching results of contour cc-1 and cc-4.